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Roll No. /

Sr. No. of Question Paper :  
 Unique Paper Code : 62357603  
 Name of the Course : B.A. (Prog.)  
 Name of the Paper : Numerical Methods  
 Semester : VI  
 Duration : 3 Hours  
 Maximum Marks : 75 Marks

**Instruction for candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.
4. Use of non-programmable scientific calculator is allowed.

1 (a) If true value is 0.0003106 and the approximate value is 0.0049065. Then find the absolute and the relative error. Differentiate between round off error and truncation error. [6]

(b) Use the Bisection method to determine the root of the equation

$$x^2 - 3 = 0$$

on the interval [1,2], perform five iterations. [6]

(c) Find a real root of the equation

$$x^3 - 5x + 1 = 0$$

on the interval (0,1) using Secant method to find the root correct up to 3 decimal places. [6]

2 (a) Apply Newton Raphson method to determine the root of the equation

$$x^3 + x^2 - 3x - 3 = 0$$

on the interval [1,2] up to four iterations. [6]

(b) Perform four iterations of the Regula-Falsi method to obtain the real root of the equation

$$x^3 - 2x^2 - 5 = 0$$

on the interval [2,3]. [6]

(c) Derive a formula for finding  $n^{\text{th}}$  root of a number N, hence find the value of  $\sqrt[3]{29}$  using Newton Raphson method. [6]

3(a) Solve the following system of linear equations using Gauss-Elimination method.

$$2x + 2y + 4z = 18$$

$$x + 3y + 2z = 13$$

$$3x + y + 3z = 14$$

(b) Find the inverse of the coefficient matrix of the system [6.5]

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

by Gauss-Jordan method with partial pivoting and solve the system. [6.5]

(c) Perform three iterations of Gauss Seidel Method for the following system of equations.

$$4x + y + z = 2$$

$$x + 3y + 2z = -6$$

Use initial approximation as  $x + 2y + 3z = -4$   
 $(x, y, z) = (0.5, -0.5, -0.5)$

[6.5]

4(a) Using the following data,

$$f(0) = 1, f(1) = 3, f(3) = 55$$

find the unique polynomial of degree 2 or less which fits the given data.

Also, obtain a bound on the error.

[6]

(b) If  $f(x) = \frac{1}{x^2}$ , find the divided difference of  $f[x_1, x_2, x_3, x_4]$

[6]

(c) Obtain the piecewise linear interpolating polynomial for the function  $f(x)$  defined by data

x	1.0	2.0	4.0	8.0
f(x)	3	7	21	73

and estimate the values of  $f(3)$  and  $f(7)$ .

[6]

5(a) Find  $f(2)$  and  $f''(2)$  using quadratic interpolation using the following data:

x	0	1	2	3
f(x)	0	1	4	3

obtain an upper bound on error also.

(b) Approximate the second order derivative of  $f(x) = e^x + 2x$  at  $x_0 = 0$ , taking  $h = 0.1, 0.01$  by using the formula

[6.5]

$$f''(x) \approx \frac{f(x_0 - h) - 2f(x_0) + f(x_0 + h)}{h^2}$$

Also approximate the derivative of  $f(x) = 1 + x + x^2 + x^3$  at  $x_0 = 0$ , taking  $h = 0.1, 0.01$  by using the formula

$$f'(x) \approx \frac{f(x_0 + h) - f(x_0 - h)}{h}$$

(c) Integrate the following function  $f(x) = \frac{1}{x}$  on the interval  $[1, 2]$ , using trapezoidal rule with 2, 4 and 8 equal subintervals.

[6.5]

6(a) Find the value of

[6.5]

$$\int_2^4 (x^2 + 1) dx$$

using Simpson's 1/3rd rule with 2 and 8 equal subintervals. Also, Find the absolute errors.

(b) Consider the initial value problem

[6.5]

$$\frac{dy}{dx} + 2y = 2 - e^{-4x}, y(0) = 1$$

find the approximate value of  $y(0.5)$  with step size  $h=0.1$  using Euler Method.

(c) Solve the initial value problem

[6.5]

$$\frac{dy}{dx} + \frac{1}{2}y = 4e^{0.8x}, y(0) = 2$$

using the Huen method over the interval  $[0, 0.4]$  with  $h = 0.1$ .

[6.5]